7.1 - Simple Subsitution

ex.: let $u = x^{1/3}$ or

let $x = u^{12}$ [when we have x's to fractional powers]

$u = \sqrt{1+\sqrt{x}}$

$u^2 = 1 + \sqrt{x}$

$\sqrt{x} = u^{1/2} - 1$

$x = (u^{1/2} - 1)^2$ [then find dx...]

7.2 - Trig Integrals

$\int \sin^m x \cos^n x\, dx$

$\int \tan^m x\, sec^n x\, dx$ [review even/odd m/n rules for these types]

7.3 - Trig Substitution

$\sqrt{a^2 - x^2} \rightarrow$ let $x = a\sin\theta$

$\sqrt{a^2 + x^2} \rightarrow$ let $x = a\tan\theta$

$\sqrt{x^2 - a^2} \rightarrow$ let $x = a\, sec\,\theta$

[Sometimes need to complete the square to get one of these forms]

Eg: x^2+6x

$= x^2 + 6x + 9 - 9$

$= (x^2 + 6x + 9) - 9$

$= (x+3)^2 - 9$ [then use simple substitution to replace x+3]

7.4 - Partial Fractions

$$\frac{1}{x^2(x^2-1)(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} + \frac{D}{(x+1)} + \frac{Ex+F}{(x^2+1)}$$

4.4 - L'hospital's Rule

Indeterminant forms when taking the Limit to infinity:

$\left[\frac{\infty}{\infty}\right]$ or $\left[\frac{0}{0}\right]$

Then apply l'Hospital's rule by taking the derivative of the numerator and denominator separately (don't use quotient rule!)

Other case: indeterminant form of $\mathbf{0^0}$ or $\mathbf{\infty^0}$ or $\mathbf{1^\infty}$:

Take the ln of the function then move the exponent down to the front of the term. Then take the limit using L'H or whatever necessary. Then 'exponentiate' the result [e^L].

7.8 - Improper Integrals

Improper integral type I:

$$\int_a^\infty f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$$

$$\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty}\int_t^b f(x)dx$$

$$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x)dx + \int_a^\infty f(x)dx$$

Improper integral type II:

If f is const on [a,b) but disconts at b, then:

$$\int_a^b f(x)dx = \lim_{t\to b^-}\int_a^t f(x)dx$$

If f is const on (a,b] but disconts at a, then:

$$\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$$

If there is a discontinuity at c s.t. a < c < b, then:

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

9.1 - Simple Ordinary Differential Equations

Initial value problem means value to sub into the function

Solving means to resolve into a dependent var and an independent var with no dy/dx

9.3 - Separable ODE's

Mixing tank problem, basic configuration:

$$\frac{d(amount)}{dt} = [\text{amount in}] - [\text{amount out}]$$

9.5 - Linear ODE's

Standard Form: $\frac{dy}{dx} + P(x)y = Q(x)$

'y' must be next to P(x), but P(x) or Q(x) can just be 1

11.1 - Sequences

- THM: Squeeze theorem for limits
- THM: Monotonic sequence thm: every bounded monotonic sequence is convergent
- THM: $\lim_{n\to\infty} r^n = \begin{cases} 0, & \text{if } -1 < r < 1 \\ 1, & r = 1 \end{cases}$

  Diverges otherwise!
- THM: if $\lim_{n\to\infty}|a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$

11.2 - 11.7 - Series

Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

Harmonic Series: $\sum_{n=1}^{\infty}\frac{1}{n} \rightarrow$diverges

P-series: $\sum_{n=1}^{\infty}\frac{1}{n^p} \rightarrow$converges for p >1

Convergence tests:

1. Divergence test [lim $a_n \neq 0$, then diverges]
2. Integral test

   Must not be alternating, must be decreasing conts function;

   Then, if the I converges, so does the series (or if D, then D)
3. Comparison test (similar to squeeze)
4. Limit Comparison test (if L exists, then C/D same as reference)
5. Alternating Series Test [$b_{n+1} < b_n$ and lim $b_n = 0$]
6. Ratio Test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| =$ L; if L < 1, then cvg; if L > 1, then dvg

11.8 - Power Series

11.9 Express Fn as Pwr Series

Start with the closed form of Geometric series

Radii of cvg is same before & after diff/integration

11.10 Taylor & Maclaurin

$$f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)(x-a)^n}{n!}$$

12.1 - 3D coordinate system

14.1 - Functions of several variables

14.2 - Limits of functions of several variables

14.3 - Partial Derivatives (multi-variable)