

Extended Euclidian Algorithm

Write $\text{GCD}(252, 198) = 18$ as a linear combination of $252 + 198$.

[Populate a table with the Euclidian method steps:]

| j | r_j | r_{j+1} | q_{j+1} | r_{j+2} | s_j | t_j | |
|-----|-------|-----------|-----------|-----------|-------|-------|--|
| 0 | 252 | 198 | 1 | 54 | 1 | 0 | } $S_0 = 1$ $t_0 = 0$ $S_1 = 0$ $t_1 = 1$ |
| 1 | 198 | 54 | 3 | 36 | 0 | 1 | |
| 2 | 54 | 36 | 1 | 18 | - | - | |
| 3 | 36 | 18 | 2 | 0 | - | - | |

At this point we can see that $\text{GCD}(252, 198)$ is indeed equal to 18.

Now fill in s + t for $j = 2, 3 + 4$.

$$s_2 = s_0 - s_1 q_1 = 1 - 0 = 1$$

$$t_2 = t_0 - t_1 q_1 = 0 - 1(1) = -1$$

$$s_3 = s_1 - s_2 q_2 = 0 - (1)(3) = -3$$

$$t_3 = t_1 - t_2 q_2 = 1 - (-1)(3) = 4$$

$$s_4 = s_2 - s_3 q_3 = 1 - (-3)(1) = 4$$

$$t_4 = t_2 - t_3 q_3 = -1 - (4)(1) = -5$$

$$S_0: 18 = 4 \cdot 252 + (-5)(198) \quad [\text{verified}]$$