

Properties of Logarithms...**Inverse trig functions, special angles...****Limit Laws**

Fundamental Laws:

- $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [c f(x)] = c \lim_{x \rightarrow a} f(x)$
- $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$

Derived Laws:

- $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ where n is positive integer
- $\lim_{x \rightarrow a} c = c$
- $\lim_{x \rightarrow a} x = a$
- $\lim_{x \rightarrow a} x^n = a^n$ where n is positive integer
- $\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$
- $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$

Squeeze theorem:**Limit Continuity:**

Continuous if:

- f(c) is defined
- The limit exists at the value of x
- f(c) is the limit L

Limit continuity theorems:

- If f and g are continuous at a and c is const, then the following functions are also continuous at a:
 - $f + g$
 - $f - g$
 - cf
 - fg
 - $\frac{f}{g}$ if $g(a) \neq 0$
- Any polynomial is conts everywhere $\mathbb{R} = (-\infty, \infty)$
 - Any rational function is conts wherever it is defined (domain).
- $\lim_{\theta \rightarrow 0} \cos \theta = 1$ $\lim_{\theta \rightarrow 0} \sin \theta = 0$
- Types of functions continuous at every number in their domain:
 - Polynomials
 - Rational functions Eg: $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$
 - Root functions
 - Trig functions
 - Inverse trig functions
 - Exponential functions
 - Log functions
- Paraphrase: Limit symbol can be moved through a function symbol if the fn is conts and the limit exists. So:

$$\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right)$$

- If g is conts at a and f is conts at g(a) then the composite function $f \circ g$ is conts at a

- 10. Intermediate value theorem.** If f is conts on the closed interval [a,b] then the conts function takes on every intermediate value between f(a) and f(b)

Derivatives of trig functions

$$\begin{aligned} (\sin x)' &= \cos x & (\cos x)' &= -\sin x \\ (\tan x)' &= \sec^2 x & (\cot x)' &= -\csc^2 x \\ (\sec x)' &= \sec x \tan x & (\csc x)' &= -\csc x \cot x \\ (a^x)' &= a^x \ln a & (e^x)' &= e^x \\ (\log_a x)' &= \frac{1}{x \ln a} & (\ln a)' &= \frac{1}{\ln a} \\ (\arcsin x)' &= \frac{1}{\sqrt{1-x^2}} & (\arccos x)' &= -\frac{1}{\sqrt{1-x^2}} \\ (\arctan x)' &= \frac{1}{1+x^2} & (\text{arccot } x)' &= -\frac{1}{1+x^2} \\ (\text{arcsec } x)' &= \frac{1}{x\sqrt{x^2-1}} & (\text{arccsc } x)' &= -\frac{1}{x\sqrt{x^2-1}} \end{aligned}$$

Slope of Tangent Line

$$y = f(x_1) + f'(x_1)(x - x_1)$$

When solving for T.L.'s and points are given instead of slope, let $x = a$ in the T.L. EQ.

Related Rates

$$\text{Volume of sphere: } V = \frac{4}{3} \pi r^3$$

$$\text{Volume of cone: } V = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of cylinder: } V = \pi r^2 h$$

$$\text{Arc of a circle: } s = r\theta$$

$$\text{Law of Cos: } a^2 = b^2 + c^2 - 2bc \cos \theta$$

4.1 Maximum and Minimum Values

- Extreme Value Thm: if f is continuous on a closed interval [a,b], then f has an absolute max and min at some values in [a,b].
- Fermat's Thm: if f has local max or min at c, and if f'(c) exists then f'(c) = 0.

Find the Critical points: where f'(c) = 0 and where f' DNE

4.2 Mean Value Theorem**Rolle's Thm:**

- f is continuous on the closed interval [a, b]
 - f is differentiable on the open interval (a, b)
 - $f(a) = f(b)$
- Then there is a number c in (a, b) s.t. $f'(c) = 0$

Mean Value Thm:

- f is continuous on closed interval [a,b]
 - f is differentiable on the open interval (a,b)
- Then there is a number c in (a,b) s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

4.8 - Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$