1 - Vectors

1.0 Vector position:

$\vec{r}_2 = \vec{V}_{avg}\Delta t + \vec{r}_1$

Momentum:

$\vec{P} = \gamma m \vec{V}$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)}}$$

Momentum, Final:

$\vec{P}_f = \vec{P}_i + \vec{F}_{NET}\Delta t$

2 - Momentum Principle:

$\Delta \vec{P} = \vec{F}_{NET}\Delta t$

Impulse:

$Impulse \equiv \vec{F}_{NET}\Delta t$

Iterative approach (use for finding 'final' positions and velocities when force is not const):

$\vec{P}_2 = \vec{P}_1 + F_{NET}\Delta t$

$\vec{P}_3 = \vec{P}_2 + F_{NET}\Delta t$

$\vec{P}_4 = \vec{P}_3 + F_{NET}\Delta t$

$\vec{P}_5 = \vec{P}_4 + F_{NET}\Delta t$

Choose the System and Surroundings

Vector Spring Force/System:

$s = |\vec{L}| - L_0$

$\vec{F} = -k_s s\,\hat{L}$

3 - Fundamental Interactions

Gravity

Electric

Strong (nuclear)

Weak

Conservation of Momentum

$\Delta P_{Sys} + \Delta P_{Surr} = 0$

Collisions

4 - Contact Interactions

Length of interatomic bonds

Stiffness of interatomic bonds (springs in series/parallel)

$$k_s = \frac{k_{si} N_{chains}}{N_{bonds}}$$

Young's Modulus: $Y \equiv \frac{stress}{strain}$

$$= \left(\frac{F_T}{A}\right) / \left(\frac{\Delta L}{L}\right)$$

Friction

Contact forces due to gasses

6 - Energy

$\Delta E_{sys} = W_{surr}$ (the energy principle)

$E_{total} = E_{REST} + k$

$E_{total} = \gamma mc^2$

$E_{REST} = mc^2$

Kinetic Energy

$k = 1/2\, mv^2$ (low speeds)

$k = \gamma mc^2 - mc^2$ (high speeds ~0.9C)

OR

$$k = \frac{P^2}{2\text{m}}$$

$$k = \frac{P^2}{(\gamma+1)m}$$

$W = \vec{F}\cdot\Delta\vec{r}$ OR

$= |\vec{F}||\Delta\vec{r}|\cos\theta$

Update form:

$E_F = E_I + W$ (the resting energy cancels from both Ef and Ei)

$\Delta U \equiv -W_i$

Multiparticle Energy Principle

$\Delta(E_1+E_2+E_3+...) + \Delta(U_{12}+U_{13}+U_{23}) = W$

$k+U<0$ is a bound state, $k+U\geq 0$ is unbounded

Minimum condition for escape: K + U = 0. So:

$$K_i + U_i = f\frac{1}{2}m v_{esc}^2 + \left(-G\frac{Mm}{R}\right) = 0$$

AND:

$k_f + U_f = k_i + U_i$

Near-earth approximation for Potential Energy

$\Delta U = mgy$

$$U_{elec} = 9\times 10^9 \frac{q_1 q_2}{r}$$

Multi-particle system where binding takes place

$Rest_F + K_F + U_F = Rest_I + K_I + U_I$

(need to consider rest energy because it changes after binding)

Power

Power (Watts) = Joules / Seconds